

Numerical Relativity meets Data Analysis: Spinning Binary Black Hole Case

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Abstract. We present a study of the gravitational waveforms from a series of spinning, equal-mass black hole binaries focusing on the harmonic content of the waves and the contribution of the individual harmonics to the signal-to-noise ratio. The gravitational waves were produced from two series of evolutions with black holes of initial spins equal in magnitude and anti-aligned with each other. In one series the magnitude of the spin is varied; while in the second, the initial angle between the black-hole spins and the orbital angular momentum varies. We also conduct a preliminary investigation into using these waveforms as templates for detecting spinning binary black holes. Since these runs are relatively short, containing about two to three orbits, merger and ringdown, we limit our study to systems of total mass $\geq 50M_{\odot}$. This choice ensures that our waveforms are present in the ground-based detector band without needing additional gravitational wave cycles. We find that while the mode contribution to the signal-to-noise ratio varies with the initial angle, the total mass of the system caused greater variations in the match.

1. Introduction

Gravitational waves produced during the coalescence of compact objects such as black holes, are one of the most promising sources for detection by interferometric detectors such as LIGO, VIRGO, GEO and TAMA [1]. For these ground based-detectors, where any gravitational-wave signals may be deeply buried in detector noise, the matched filtering technique [2] is the detection strategy of choice. Matched filtering is optimal when accurate representations of the expected signal are used. For low mass compact object binaries that means templates built from well-known analytic methods such as the post-Newtonian (PN) approximation [3]. When the total mass is larger than approximately $50M_{\odot}$ [4], the merger of the binary black hole (BBH) system will not only be present in the sensitivity band of ground-based detectors but also generate the strongest signal. Quantifying the last orbits and merger of a BBH system has long been the purview of numerical relativity.

A new direction in the efforts to generate accurate templates is the inclusion of waveforms produced by numerical relativity. The waveforms from numerical relativity may be used in data analysis methods involving searches for inspiraling sources

in several ways. For instance, they can be used in creating hybrid templates for detection, validating and extending the mass range of the current search strategy and may be implemented as templates themselves especially for the higher mass range [5, 6, 7, 8, 9, 10, 11].

In a previous paper [9] (paper I), we studied waveforms from a series of numerical evolutions of equal-mass, spinning BBH mergers, which we label the A-series. In the A-series, the black holes had spins that were equal in magnitude with one spin aligned and the other anti-aligned with the orbital angular momentum, J_{orb} . We found that using only the dominant harmonic of the radiation, a common practice in numerical relativity and data analysis, caused a complete degeneracy of the this spin space with respect to detection. In other words, a template of a non-spinning BBH waveform faithfully matched the signal of any anti-aligned spinning waveform when only the dominant modes were used. However, retaining non-dominant harmonics of non-negligible amplitude broke the degeneracy to some degree, most notably for spins $J/M^2 \geq 0.6$. In this paper, we further our previous study in two ways. First we include a new series of BBH mergers, the B-series, in which the initial black-hole spins are still equal in magnitude and anti-aligned with each other but the initial angle the black-hole spins make with the orbital angular momentum, ϑ , is allowed to vary. Second, for both the A and B series of data we conduct a new study of the contributions of the individual modes to the signal-to-noise ratio. In addition, for the B-series we also calculate the dependence of the minimax matches between the dominant mode and the full waveform on the initial spin orientation.

2. The Waveforms

The binary black hole coalescence problem can be divided into three phases called the inspiral, merger and ringdown. Due to the early accessibility of PN and other analytic based approaches, most of the work in setting up detection schemes has been done with these analytic approaches. These methods are well suited for the inspiral phase of the coalescence, only breaking down at some yet-to-be-determined point within a few orbits of the merger. Fortunately, the signal resulting from the inspiral is in the ground-based frequency band for systems of total mass less than approximately $50M_{\odot}$. As the mass increases, the inspiral lowers in frequency, and the detectable signal contains more merger and ringdown. Now that numerical relativity is producing the waveforms for the final orbits and the merger phase of the coalescence [12, 13, 14] we can investigate the inclusion of the merger regime in detection strategies.

We study two sets of waveforms both the result of evolutions conducted by the PSU numerical relativity group. The A-series was published in [15] with initial black-hole spins covering the set $a = 0.0, 0.2, 0.4, 0.6, 0.8$. The B-series was published in [16] and generalizes the A-series with variation of the initial angle that one of the anti-aligned spins makes with the axis, ϑ , at a fixed magnitude of spins, $a = 0.6$. When $\vartheta = 0$, we recover the $a=0.6$ waveform of A-series. When $\vartheta = \pi/2$, the spin-directions lie in the

plane of the orbit and are in the “superkick” [17] configuration in which the maximum gravitational recoil from the BBH mergers has been found.

Since these waveforms were originally produced to study the gravitational recoil imparted to the final black hole after an asymmetric collision, only two to three orbits were evolved (the merger phase dominates the recoil). The number of orbits is set by the initial orbital frequency for a given total mass. In order to place the numerical waveforms firmly in the frequency band of the detector, we use the initial LIGO noise curve [18] and we only investigate masses larger than $50M_\odot$ when calculating matches between waveforms. The total mass sets the frequency at which the signal enters the band. For example, the cutoff frequency for a binary system of $50M_\odot$ is $0.02/M$ or approximately 80Hz and about 40Hz for $100M_\odot$.

The waveforms were extracted from the numerical evolution of the spacetime in terms of the Newman-Penrose scalar, $\Psi_4(t, x, y, z)$, which is expanded into angular modes via ${}_{-2}Y_{\ell m}(\theta, \phi)$, the spin-weighted $s = -2$ spherical harmonics, by extraction on a sphere. The dominant mode for the quasi-circular orbits is the quadrupole mode ($\ell = m = 2$). The angles θ and ϕ correspond to the inclination and azimuthal angles between the source and detector in the source frame. When $\theta = 0$, the observer is directly above the orbital plane of the binary and sees primarily the $\ell = m = 2$ mode. As θ increases, the waveforms are a mixture of modes. We truncate the infinite series of modes at $\ell \leq 4$ because we do not extract all the modes from the simulations and modes of $\ell > 4$ were zero within our numerical error. As more complicated configurations are evolved, more modes will need to be accurately extracted from the codes.

3. Faithfulness

The multipolar analysis of BBH waveforms produced by numerical relativity has been pursued for both unequal mass and spinning BBH configurations [19, 20, 21]. In paper I, we found that using only the dominant mode in comparing waveforms, tantamount to choosing an inclination angle with the detector in the source frame of $\theta = 0$, resulted in a degeneracy of the A-series parameter space. In this paper, we focus our attention on how different initial configurations, in this case a and ϑ , result in different mode contributions to the signal-to-noise ratio. To build intuition about what parameters might be important to the template space of black-hole mergers, we further calculate the overlap between pairs of our waveforms. We perform a preliminary analysis on how faithful $\ell = m = 2$ waveforms of various parameters would be in matching with waveforms at random inclination angle. We keep the total mass for each template fixed and vary the inclination angle of the detector, θ , the spin a , and initial angle ϑ when appropriate.

The minimax match is given by [22, 23]

$$M \equiv \max_{t_0} \min_{\Phi} \frac{\langle h_1 | h_2 \rangle}{\sqrt{\langle h_1 | h_1 \rangle \langle h_2 | h_2 \rangle}}, \quad (1)$$

where

$$\langle h_1 | h_2 \rangle = 4 \operatorname{Re} \int_{f_{\min}}^{f_{\max}} \frac{\tilde{h}_1(f) \tilde{h}_2^*(f)}{S_h(f)} df. \quad (2)$$

The Fourier transform of the strain, $h(t)$, is given by $\tilde{h}_+(f) = \mathcal{F}(\operatorname{Re}(\Psi_4))(f)/(-4\pi^2 f^2)$ where $\Psi_4(t) = \frac{d^2}{dt^2}(h_+(t) - ih_\times(t))$, and \mathcal{F} is a Fast Fourier Transform. The signal-to-noise ratio, ρ , is given by

$$\rho = \left[4 \int_{f_{\min}}^{f_{\max}} \frac{|\tilde{h}(f)|^2}{S_h(f)} \right]^{1/2}. \quad (3)$$

The variable $S_h(f)$ denotes the noise spectrum for which we use the initial LIGO noise curve. The domain $[f_{\min}, f_{\max}]$ is determined by the detector bandwidth and the masses of our signal. The masses are set such that the overlaps will not change significantly if we were to add the inspiral portion of the signal because the A and B series of waveforms have orbital frequencies that increase almost monotonically. Owing to this, the gravitational wave frequency also increases monotonically with time implying that extending the signal back in time will not change the spectrum in the merger band. When precession is significant this will no longer be true and the inspiral will likely contribute to the signal at higher frequencies.

A-series: In Fig. 1 we plot the match versus spin at different inclination angles, θ , for a given total mass of $100M_\odot$. This plot first appeared in paper I and is included here for reference. The figure shows that as the spin increases, the match between a waveform of $\theta = 0$ and one of non-zero θ decreases. This indicates that the non-dominant modes are important both for distinguishing between different spinning waveforms and in making a detection. This is most notable for the $a = 0.8$ case.

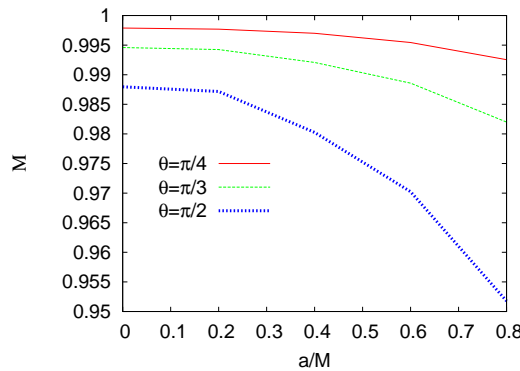


Figure 1. The minimax match versus a for three values of θ , $\pi/4, \pi/3, \pi/2$ for the case of total mass $100M_\odot$. The value of $\theta = 0$ is not included in the plot, since this is the waveform of comparison and the match is one by definition.

In order to have a qualitative understanding of why the match behaves as in Fig. 1, we study the ρ per mode for the A-series. This is only a qualitative estimate because the relative fraction of modes present in the signal will depend on the relative spin-weighted spherical harmonics values at the particular angle. In practice the error induced by

ignoring the mixed terms that are important in constructing $\rho(\theta, \phi)$ from the modes is less than 20%, as the relative overlaps of the significant modes from both the A and B-series of data are of this order. Since the ρ of the $\ell = m = 2$ mode is much larger than the ρ of the other modes, we plot the ratio, $\rho(\ell, m)/\rho(2, 2)$ in Fig. 2. The upper left plot corresponds to a system of mass $50M_\odot$, the upper right to $100M_\odot$, lower left to $200M_\odot$ and lower right to $300M_\odot$. Across all the masses sampled, the ratio of the ρ for

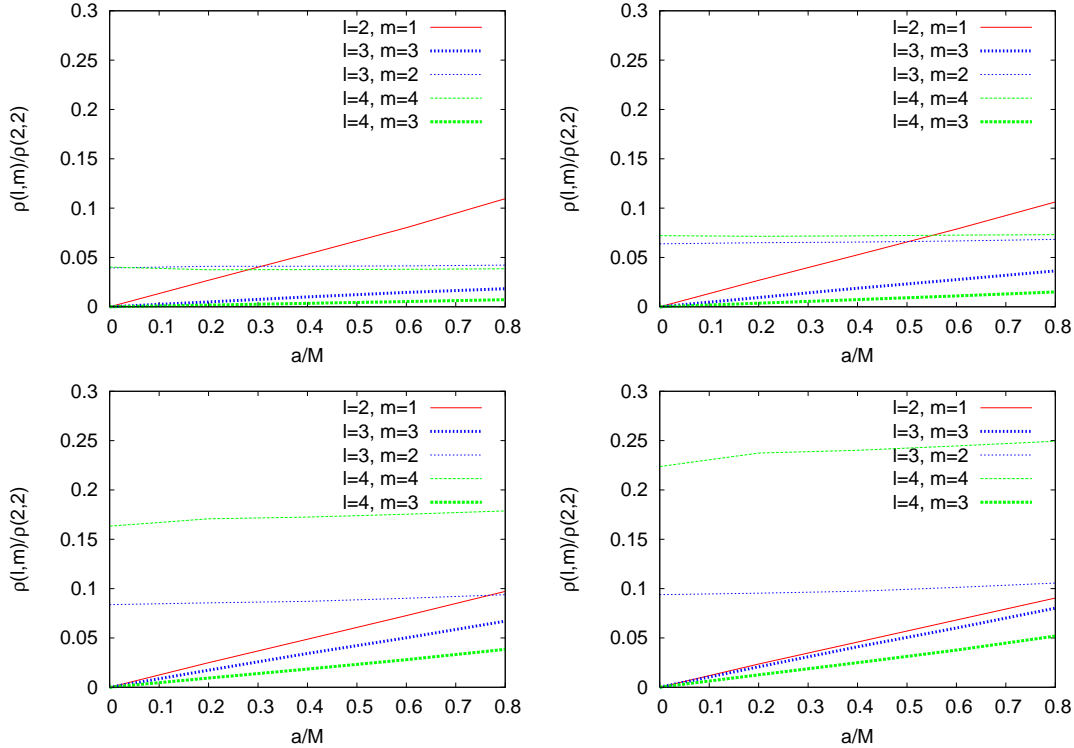


Figure 2. A series of plots are shown with the ratio of ρ per mode to the ρ of the $\ell = m = 2$ mode versus the initial spin of the black holes. This is done for series-A. Each plot refers to the calculation for a different total mass of the binary. Starting from the upper left and moving right and then down, we have $50M_\odot$, $100M_\odot$, $200M_\odot$, and $300M_\odot$ on the lower right.

each mode grows with increasing spin. This is especially true for the $m = 1$ and $m = 3$ modes which are suppressed at low a . The $m = 2$ and $m = 4$ increase slightly with a . While the $\ell = 2, m = 1$ mode is the next mode dominant mode after the $\ell = m = 2$ mode for the high-spin regime at low masses, the $\ell = m = 4$ mode is secondary for the entire a range at higher masses. For low spins, the waveform is entirely dominated by the $m = 2$ modes. The linear-like growth of the odd- m modes with spin magnitude is expected from PN expressions like Eq(1)-(4) in [21].

B-series: The minimax match versus the initial angle for the B-series is presented in Fig. 3. Each line represents a choice of total mass, with $50M_\odot$ the top most line and $300M_\odot$ the bottommost. The match was computed by setting one waveform to $\theta = 0$ and the other to $\theta = \pi/2.4$. We find that the variation of the match across initial angle for the given spin of $a = 0.6$ does not change more than about 2%. The variation

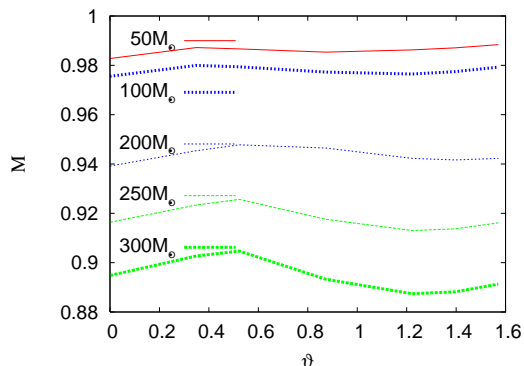


Figure 3. The minimax match versus initial angle ϑ for the B-series. Each curve represents a particular choice of total mass, $50M_{\odot}$ at the top with each successively lower line a higher mass. To compute the match, we used one waveform with $\theta = 0$ and the other at $\theta = \pi/2.4$ radians for a given a and ϑ .

amongst different total masses is more dramatic, dropping down below a match of 0.9 for most in the angles at a mass of $300M_{\odot}$. At that large mass range, the ringdown is contributing significantly to the signal, and differences in the modes, like the $\ell = m = 4$ mode, begin to make important contributions. These BBH configurations settle down to a final black hole with a spin of $a = 0.62$.

In Fig. 4, we once again investigate a qualitative interpretation of the match through the ρ as plotted versus the initial angle, ϑ for each mode. The upper left plot corresponds to a system of mass $50M_{\odot}$, the upper right to $100M_{\odot}$, lower left to $200M_{\odot}$ and lower right to $300M_{\odot}$. Across the mass scales sampled, as ϑ increases, the signal in odd- m modes decrease. In the non-precessing case, the strength of the odd-modes is expected to vary with the z-component of the spin [21]. Since in this series of runs, the spins precess about the z-axis and ϑ remains nearly constant, the relative strength of the modes show similar trends as the non-precessing case. At higher masses, where the ringdown dominates the signal, the $\ell = m = 4$ mode contributes a large portion of the ρ of the total signal, over 20% for a BBH of $300M_{\odot}$. This is in part due to the enhancement of the ringdown signal which occurs when the frequency of the higher modes lies around the detector's sweet spot. It is interesting to note that in the "superkick" the spread of the modes is reduced in ρ compared with the parallel configuration at $\vartheta = 0$. The decrease of the odd m modes is expected from the PN expressions. For example, the $\ell = 2, m = 1$ will be suppressed when the spins lie in the orbital plane for equal-mass black holes as discussed in [21].

These waveforms are the solution to the BBH coalescence as expressed by general relativity with errors arising from several sources, see [24]. In paper I, we analyzed the effects of resolution on the matched filtering technique and found that for the resolutions used to compute the waveforms studied in the A-series, the largest error would be ± 0.02 in the match, although that is only for the $a = 0.8$ case, and is typically smaller. The waveforms in the B-series have comparable errors, i.e. the resolutions, wave extraction

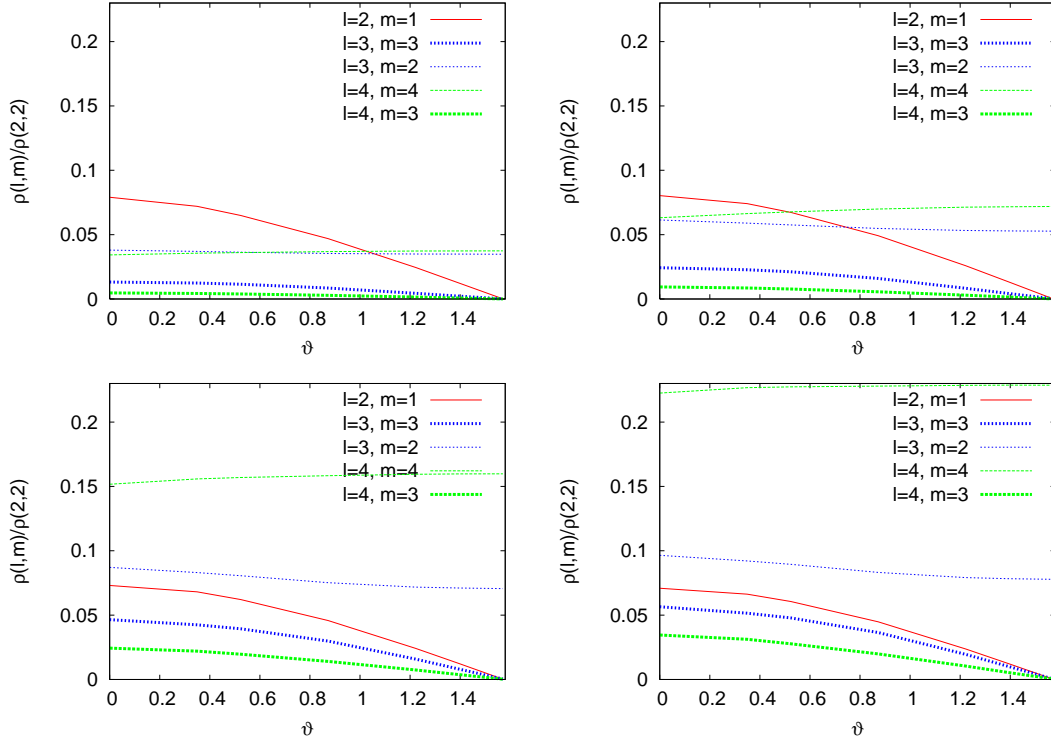


Figure 4. A set of plots is shown with the ratio of ρ per mode to the ρ of the $\ell = m = 2$ mode versus the initial angle for series-B. Each plot is the ratio computed for a different total mass of the binary. Starting from the upper left and moving right and then lower left and right, we have $50M_{\odot}$, $100M_{\odot}$, $200M_{\odot}$, and $300M_{\odot}$ on the lower right.

and other numerical techniques were the same in computing both series of waveforms as discussed in [9, 15, 16]. For reference, the typical resolution on the finest grids were $h = M/35.2$ where M is the total mass.

4. Discussion and Conclusion

In this paper, we investigated the contribution of individual modes to ρ from the last orbits, merger and ringdown of an equal-mass, spinning binary black coalescence. In the A-series, the spins were kept parallel/anti-parallel to the direction of the orbital angular momentum, but the magnitude of the spins varied. In the B-series, the magnitude was kept fixed to $a = 0.6$, but the initial angle the spins make with the orbital angular momentum varied.

In paper I, we investigated the match between a waveform from the A-series containing only the $\ell = m = 2$ mode and a waveform of a sum of modes. There we found strong dependence on the match with spin, with the $\ell = m = 2$ waveform failing to match to spinning waveforms especially for spins equal to and greater than $a = 0.6$. We did a similar study here for the B-series, comparing two waveforms of $a = 0.6$ at various ϑ . We found, despite the variation of the ρ versus ϑ , the match had a much

greater dependence on mass than the initial angle. The inclusion of modes was much more important to templates of higher mass, where the merger and ringdown dominate the signal, than at lower masses. This importance will be more evident in matches using unequal-mass and spinning waveforms with larger spin as well as waveforms with more cycles.

To qualitatively understand the matches, we conducted a multipolar analysis of the modes in each waveform and calculated the ratio of the ρ of each mode versus $\ell = m = 2$. For the A-series, the ρ per mode increased as the magnitude of the spins increased at every total mass as seen in Fig. 2. The odd- m modes increased from almost no contribution at low spins to a 10% contribution at larger spins. At a given spin, the $\ell = 2, m = 1$ mode dominated the ρ at low mass, but the $\ell = m = 4$ mode's ratio to $\ell = m = 2$ grew with increasing mass. For the B-series, Fig. 4, we found that the diversity of contributing modes decreases with increasing angle, except for the $\ell = m = 4$ and $\ell = 3, m = 2$ modes which remain relatively constant across ϑ for a given mass. As in the variation with a , at low total binary mass, the secondary signal is the $\ell = 2, m = 1$ mode, but at higher masses the $\ell = m = 4$ and the $\ell = m = 2$ modes are stronger. As anticipated, the $\ell = 2, m = 1$ mode decreases to zero as the initial angle moves to lie parallel to the orbital plane. For both the series of runs, the variation of the signal in different modes is consistent with the expectation from PN [21].

5. Acknowledgments

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